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TOPIC PLAN		
Partner organization	"Goce Delcev "- University Shtip, North Macadonia	
Topic	Differentiation	
Lesson title	Minimizing Material: Surface Area	
Learning objectives	The student will: • calculate surface area and volume of a cylinder • use technology (Excel spreadsheet, GeoGebra) to test and judge optimized dimensions • use first derivative to find extreme value • use second derivative to find maximum or minimal value	**Strategies/Activities** ☐Graphic Organizer ☐Think/Pair/Share ☐Modeling ☑Collaborative learning ☑ Discussion questions ☑Project based learning ☑Problem based learning **Assessment for learning** ☑Observations ☑Conversations ☑Work sample ☐Conference ☐Check list ☐Diagnostics **Assessment as learning** ☐Self-assessment ☑Peer-assessment ☑Presentation ☐Graphic Organizer ☑Homework
Aim of the lecture / Description of the practical problem	A manufacturer of food-storage containers wants to make a cylindrical can with a volume of 1000 cm^3. The manufacturer wants the cost of can is as low as possible. For this problem, we need to find the dimensions (radius and height of cylinder) such that the surface area is a small as possible.	
Previous knowledge assumed:	The student needs to know: • to calculate volume and surface area of cylinder. • to calculate first and second derivates • to know differentiation Techniques: The Power and Sum–Difference Rules • to know differentiation Techniques: The Product and Quotient Rules	

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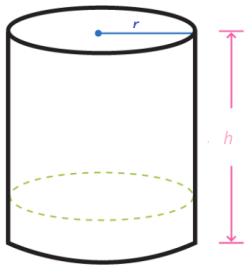
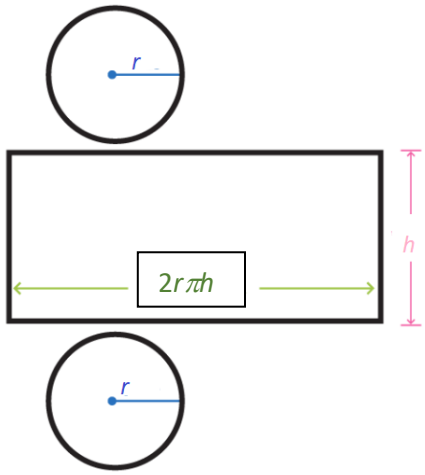
Introduction / Theoretical basics	A cylinder is a 3D figure that has two circucar faces, one at the top and one at the bottom, and one curved surface. A cylinder has a height *h* and a radius *r*. Volume of cylinder is calculate with formula: $V = \pi r^2 h$ The cylinder consists of two circles and one rectangle. The area of the circles is $r^2\pi$. One side of the rectangle is height *h*, the other side of the rectangle is equal to perimeter of the circle, $2r\pi$. So, the area of the rectangle is $2r\pi$.  $V = \pi r^2 h$  $Surface\,area\,(A) = 2r^2\pi + 2r\pi h$	**Assessment of learning** ☑Test ☑Quiz ☑Presentation ☑Project ☐Published work

For a function $y = f(x)$ its derivative at x is the function f' defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided that the limit exists. If $f'(x)$ exists, then we say that f is differentiable at x.

The calculation of derivate of function is performed by help of the table for derivates of elementary functions and by rules for calculation of derivates of functions that are not elementary.

The derivates of some elementary function are:

$$f(x) = c, \quad f'(x) = 0$$

$$f(x) = x^n, \quad f'(x) = nx^{n-1}$$

$$f(x) = \frac{1}{x}, \quad f'(x) = -\frac{1}{x^2}$$

$$f(x) = \sqrt{x}, \quad f'(x) = \frac{1}{2\sqrt{x}}$$

$$f(x) = e^x, \quad f'(x) = e^x$$

$$f(x) = \ln x, \quad f'(x) = \frac{1}{x}$$

$$f(x) = a^x, \quad f'(x) = ax^{n-1}$$

Let $f(x)$ and $g(x)$ are differentiable functions and c is a constant. Then each of the following equations holds.

$(cf(x))' = cf'(x)$ - constant multiple rule

	$\left(f(x)+g(x)\right)' = f'(x)+g'(x)$ - sum rule $\left(f(x)-g(x)\right)' = f'(x)-g'(x)$ - difference rule $\left(f(x)\cdot g(x)\right)' = f'(x)\cdot g(x)+f(x)\cdot g'(x)$ - product rule $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{\left(g(x)\right)^2}$ for $g(x)\neq 0$ - quotient rule The derivates are used for finding extreme value of the function. The function f has a minimum value at $x=c$ if $f'(c)=0$ and $f''(c)>0$. The function f has a maximum value at $x=c$ if $f'(c)=0$ and $f''(c)<0$..	
Action	Let *h* is a height of the cylinder and *r* is a radius (both measured in centimeters). Volume of cylinder is $V=\pi r^2 h$ From a formula of volume, we can find relate between r and h. The height can be expressed through the radius as follows: $\pi r^2 h = 1000$ $h=\frac{1000}{\pi r^2}$. The surface area of cylinder is sum from area of two circle ($r^2\pi$) and area of rectangle ($2r\pi h$) :	

$$A = 2r^2\pi + 2r\pi h$$

With substituting for *h* is obtained:

$$A = 2r^2\pi + 2r\pi\frac{1000}{\pi r^2} = 2r^2\pi + \frac{2000}{r}$$

Now, area *A* is function only of radius *r*.

$$A(r) = 2r^2\pi + \frac{2000}{r}$$

It is clear, that the radius *r* is greater to zero (*r* > 0) .

By differentiation is obtained:

$$A'(r) = 4r\pi - \frac{2000}{r^2}$$

To find a extreme value, we set the derivative equal to 0.

$$4r\pi - \frac{2000}{r^2} = 0$$

$$4r\pi = \frac{2000}{r^2}$$

$$r^3 = \frac{2000}{4\pi}$$

$$r^3 = \frac{500}{\pi}$$

$$r = \sqrt[3]{\frac{500}{\pi}} \approx 5.41.$$

The critical value $r \approx 5.41$ is the only critical value in the interval (0, ∞).
To find that the critical value is a minimal or maximal value, we will find the second derivative:

$$A''(r) = 4\pi + \frac{4000}{r^3}..$$

With substituting the value of the radius r = 5.41

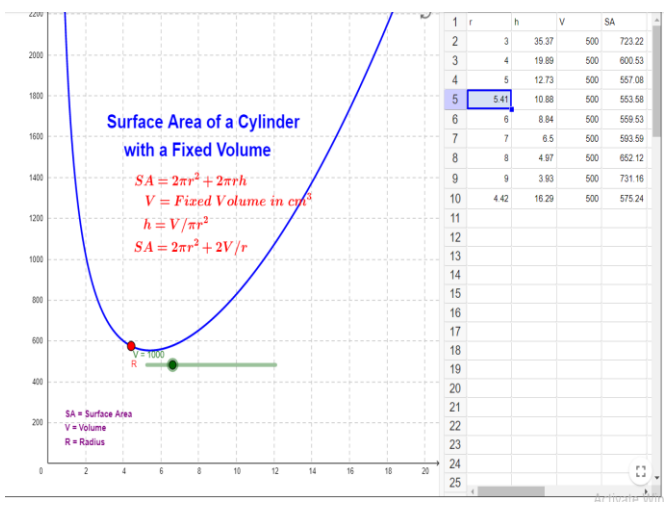
$$A''(r) = 4\pi + \frac{4000}{(5.41)^3} > 0$$

The graph is concave up at the critical value 5.41, so the critical value is a minimum value. Thus, the radius should be approximately 5. 41. The height $h = \frac{1000}{\pi r^2} = \frac{1000}{\pi (5,41)^2} \approx\sim 10.88.$

Minimum total surface area approximately is

$$A = 2r^2\pi + 2r\pi h \approx 553.58$$

We can use the software GeoGebra to represent the relation between radius r and surface area A for fixed volume. Also, from the graphic we can see minimal value that for surface area can be obtained for fixed volume.



	r	h	V	SA
2	3	35.37	500	723.22
3	4	19.89	500	600.53
4	5	12.73	500	557.08
5	5.41	10.88	500	553.58
6	6	8.84	500	559.53
7	7	6.5	500	593.59
8	8	4.97	500	652.12
9	9	3.93	500	731.16
10	4.42	16.29	500	575.24

Materials / equipment / digital tools / software	The materials for learning are given as a part of references of the end from this topic plan; Equipment: classroom, green board, chalk in different colors; Digital tools: laptop, projector, smart board; Software: Mathematica, GeoGebra. inTexna .	
Consolidation	The students through the above example should understand that derivatives can be are used to solve many problems in real life. Also, that the first derivatives are used to find extreme values, and with the second derivatives it is found whether this value is minimum or maximum. Give the similar problem of the students, but instead of can to have shape of cylinder, to have shape of cubioid with square cross section. Question to the students: • What difference would it make to the surface area if a cuboid with square cross section was used to hold the 1000 cm3 of drink? • Do you think a cylinder is the best shape to use? Why?	

Reflections and next steps	
Activities that worked	**Parts to be revisited**
After the class, the teacher according to his personal perceptions regarding the success of the class fills in this part.	Through the success of the homework done by the students, questions and discussion at the beginning of the next class, the teacher comes to the conclusion which parts of this class should be revised.

References

[1] M. L. Bittinger, D. J. Ellenbogen and S.A. Surgent (2012), “Calculus and its applications”, Addison-Wesley
[2] G. Strang “Calculus” , Wellelye-Cambridge Press
[3] S. Calaway D. Hoffman and D.Lippman (2014) “Applied Calculus”
[4] P.D. Lax, M. S.Terrell (2014) “Calculus with Applications”, Springer